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HEAT EXCHANGE OF A CYLINDER WITH LOW-FREQUENCY OSCILLATIONS

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It is well known that the presence of a sonic field intensifies heat-mass exchange processes [1-3], and that this intensification is due to the presence of stationary secondary flows formed near the solid surface [1]. However, existing theoretical treatments of this question are limited to the case of high-frequency oscillations, while the situation in which the thickness of the Stokes layer is comparable to or larger than the size of the body is no less important. For example, such a situation is realized in heating devices operating in a high-frequency instability regime and using atomized liquid or solid fuel. These problems are of importance in thermoanemometry. In the present study the example of a circular cylinder will be used to study the effect of low-frequency oscillations on local and integral characteristics of the heat exchange process. By low frequency, we refer to the region where the Stokes layer thickness [$\delta_{ac} \sim (\nu/\omega)^{0.5}$] is comparable to or larger than the cylinder size.

Let a circular cylinder of radius a and infinite length be located within an infinite viscous liquid, which at an infinite distance from the cylinder undergoes oscillations following a harmonic law with cyclical frequency ω. The temperatures of the cylinder surface $\tilde{T}_W$ and the surrounding medium $\tilde{T}_\infty$ are considered constant, and the temperature difference $(\tilde{T}_W - \tilde{T}_\infty)$ is assumed so small that changes in the physical properties of the liquid and natural convection may be neglected. Also neglecting dissipative effects, we write the energy equation in the form [3]:

$$\frac{\partial T}{\partial \tau} + \frac{\varepsilon}{1+r}\frac{\partial(\psi, T)}{\partial(r, \theta)} = \frac{H^2}{\Pr}\nabla^2 T \tag{1}$$

with boundary conditions

$$T = 1 \text{ for } r = 0, \ T = 0 \text{ for } r \to \infty. \tag{2}$$

The dimensionless quantities in Eqs. (1), (2) are defined as follows:

$$r = (\tilde{r} - a)/a, \ \psi = \tilde{\psi}/Ba, \ \tau = \tilde{\tau}\omega, \ T = (\tilde{T} - \tilde{T}_\infty)/(\tilde{T}_w - \tilde{T}_\infty),$$

where $\varepsilon = S/a$; $H = \delta_{ac}/a$; $\delta_{ac} = \sqrt{\nu/\omega}$; S is the amplitude of the acoustical displacement of the medium; $B = S\omega$ is the amplitude of the velocity pulsations. The tilda superscript denotes quantities having dimensions.

We will consider the case in which $\varepsilon \ll 1$ (a similar assumption was used in solving the hydrodynamic portion of the problem [4]). Then, using the perturbation method, we write the solution of Eq. (1) in the form of a series

$$T = T_0 + \varepsilon T_1 + O(\varepsilon^2) \tag{3}$$

and similarly represent the velocity field

$$\psi = \psi_0 + \varepsilon\psi_1 + O(\varepsilon^2). \tag{4}$$

We recall that according to [4], ψ_0 is a periodic function of time with frequency ω and contains no time-independent component, while ψ_1 consists of two components, a stationary ψ_1^{st} and a periodic ψ_1^{p}, which varies with a cyclical frequency 2ω.

Since we are interested in the effect of low-frequency oscillations on the heat exchange of the circular cylinder, we will assume further that $H = O(1)$.

We will consider the case where $\Pr = O(1)$. Substituting Eqs. (3), (4) in Eq. (1) and collecting terms with identical powers of ε, we obtain the following equations:

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$$\frac{\partial T_0}{\partial \tau} = \frac{H^2}{\Pr} \nabla^2 T_0; \tag{5a}$$

$$\frac{\partial T_1}{\partial \tau} + \frac{1}{1+r} \frac{\partial (\psi_0, T_0)}{\partial (r, \theta)} = \frac{H^2}{\Pr} \nabla^2 T_1. \tag{5b}$$

We will consider Eq. (5a). We write T_0 as the sum of stationary and pulsating components

$$T_0 = T_0^{st} + T_0^{p}$$

and write separately equations for each component

$$\frac{\partial T_0^{p}}{\partial \tau} = \frac{H^2}{\Pr} \nabla^2 T_0^{p}; \tag{6a}$$

$$\nabla^2 T_0^{st} = 0. \tag{6b}$$

Since in boundary conditions (2) there is no time dependence, then $T_0^{p} \equiv 0$. Consequently, the temperature T_0 is independent of time, and as follows from Eq. (6b), the heat liberation process is determined by thermal conductivity alone (in this case for the cylinder Nu $\equiv 0$). We are interested in the situation in which the heat liberation process with low frequency oscillations is convective, i.e., is determined by the structure of secondary flows formed in the oscillations. In this case, the Prandtl number Pr having increased, it is necessary to decrease the contribution of conductive terms to the heat transfer process.

We will consider the case where $P = O(\varepsilon^{-1})$, i.e., $(\varepsilon \Pr) = O(1)$. Then, substituting Eqs. (3) and (4) in Eq. (1) and collecting terms with like powers of ε, we obtain, considering that $(\varepsilon \Pr) = O(1)$,

$$\partial T_0 / \partial \tau = 0; \tag{7a}$$

$$\frac{\partial T_1}{\partial \tau} + \frac{1}{1+r} \frac{\partial (\psi_0, T_0)}{\partial (r, \theta)} = \frac{H^2}{\varepsilon \Pr} \nabla^2 T_0. \tag{7b}$$

It follows from Eq. (7a) that as in the case $\Pr = O(1)$, to the accuracy of terms of order ε the temperature is time-independent. Considering this fact, we will average Eq. (7b) over the oscillation period. Then, keeping in mind that $\psi_0 \sim \cos \tau$, we obtain $\nabla^2 T_0 = 0$, i.e., for moderately large Prandtl numbers the process of heat exchange with low frequency oscillations is also conductive.

We will consider the case $(\varepsilon^2 \Pr) = O(1)$. Using Eqs. (1), (3), (4) and repeating the same procedure used in deriving Eq. (7), we obtain, to the accuracy of terms of order ε^2,

$$\partial T_0 / \partial \tau = 0; \tag{8a}$$

$$\frac{\partial T_1}{\partial \tau} + \frac{1}{1+r} \frac{\partial (\psi_0, T_0)}{\partial (r, \theta)} = 0; \tag{8b}$$

$$\frac{\partial T_2}{\partial \tau} + \frac{1}{1+r} \left[\frac{\partial (\psi_1, T_0)}{\partial (r, \theta)} + \frac{\partial (\psi_0, T_1)}{\partial (r, \theta)} \right] = \frac{H^2}{\varepsilon^2 \Pr} \nabla^2 T_0. \tag{8c}$$

We will consider Eq. (8b). Since T_0 is time independent, while $\psi_0 \sim \cos \tau$, then

$$T_1 = T_{10}(r, \theta) + T_{11}(r, \theta) \sin \tau.$$

Then, averaging Eq. (8c) over the oscillation period, we obtain

$$\left\langle \frac{1}{1+r} \left[\frac{\partial (\psi_1, T_0)}{\partial (r, \theta)} + \frac{\partial (\psi_0, T_1)}{\partial (r, \theta)} \right] \right\rangle = \frac{H^2}{\varepsilon^2 \Pr} \nabla^2 T_0. \tag{9}$$

The second term on the left side of Eq. (9) is equal to zero, since the functions describing the time dependence of ψ_0 and T_1 are orthogonal over the interval $[0, 2\pi]$. Considering this fact, we write Eq. (9) in the form

$$\frac{1}{1+r} \frac{\partial (\psi_1^{st}, T_0)}{\partial (r, \theta)} = \frac{H^2}{\varepsilon^2 \Pr} \nabla^2 T_0, \tag{10}$$

where ψ_1^{st} is the stationary component of the flow function.

Thus, for low-frequency oscillations convective heat transfer becomes significant only in the case of large Prandtl numbers $\Pr = O(\varepsilon^{-2})$. Moreover, as follows from Eq. (10), the pulsation components of the velocity and temperature prove to have no effect on the stationary temperature field. A similar result was obtained for the case of high-frequency oscillations in [1].

From the structure of Eq. (10) it also follows that in the case where $(\varepsilon^2 \Pr) \gg 1$, near the cylinder surface there will be formed a thermal boundary layer, the thickness of which [considering that near the surface

$\psi_1^{st} \sim r^2$, and also $H = O(1)$] is of the order of $O[a(\varepsilon^2 \mathrm{Pr})^{-1/3}]$. Then, introducing the variables corresponding to the thermal boundary layer,

$$y = k^{-1} r, \quad t_0(y, \theta) = T_0(r, \theta), \quad k = (\varepsilon^2 \,\mathrm{Pr})^{-1/3},$$

and also expanding the flow function ψ_1^{st} and temperature in a series in k

$$\psi_1^{st} = k^2 \beta_1 + k^3 \beta_2 + O(k^4), \quad t_0 = t_{00} + k t_{01} + O(k^2)$$

and substituting in Eq. (10) we obtain, limiting ourselves to terms first order in k:

$$\frac{\partial \beta_1}{\partial y} \frac{\partial t_{00}}{\partial \theta} - \frac{\partial \beta_1}{\partial \theta} \frac{\partial t_{00}}{\partial y} = H^2 \frac{\partial^2 t_{00}}{\partial y^2}, \tag{11}$$

where

$$\beta_1 = \frac{1}{2} y^2 \left(\frac{\partial^2 \psi_1^{st}}{\partial y^2} \right)_{y=0}; \quad \beta_2 = \frac{1}{6} y^3 \left(\frac{\partial^3 \psi_1^{st}}{\partial y^3} \right)_{y=0}.$$

We will now determine the explicit form of the function β_1. To do this we use the solution of the hydrodynamic section of the problem, presented in [4]:

$$\psi_1^{st} = \frac{1}{4\Delta} \Big\{ \gamma \frac{a^2}{\tilde{r}^2} \left[\mathrm{ker}_2^{(')} \gamma \,\mathrm{kei}_0 \gamma - \mathrm{kei}_2^{(')} \gamma \,\mathrm{ker}_0 \gamma \right] + \gamma \left[\mathrm{kei}_0 \gamma \,\mathrm{ker}_0^{(')} \gamma - \mathrm{ker}_0 \gamma \,\mathrm{kei}_0^{(')} \gamma \right]$$

$$+ 2 \left[\mathrm{ker}_2 \left(\frac{\tilde{r}^2}{a^2} \right) \mathrm{kei}_0 \gamma - \mathrm{kei}_2 \left(\frac{\tilde{r}^2}{a^2} \right) \mathrm{ker}_0 \gamma \right] \Big\} \sin 2\theta, \tag{12}$$

where $\Delta = \mathrm{ker}_0^2 \gamma + \mathrm{kei}_0^2 \gamma$; $\mathrm{ker}_2^{(')} x = d/dx \,\mathrm{ker}_2 x$; $\gamma = H^{-1}$; $\mathrm{ker}_n x$, $\mathrm{kei}_n x$ are Thompson functions.

Using Eq. (12), we obtain

$$\beta_1 = \frac{1}{4H^2} y^2 \sin 2\theta.$$

Then we rewrite Eq. (11) in the form

$$y \sin \varphi \frac{\partial t_{00}}{\partial \varphi} - \frac{1}{2} y^2 \cos \varphi \frac{\partial t_{00}}{\partial y} = H^4 \frac{\partial^2 t_{00}}{\partial y^2}, \tag{13}$$

where $\varphi = 2\theta$ and is measured from a line coinciding with the direction of cylinder oscillations. Introducing the variable

$$x = \left(\frac{1}{9H^4} \right)^{1/3} y \frac{\sin^{1/2} \varphi}{\left[\int_0^{\varphi} \sin^{1/2} \chi d\chi \right]^{1/3}},$$

we reduce Eq. (13) to an ordinary differential equation

$$\frac{d^2 t_{00}}{dx^2} + 3x^2 \frac{dt_{00}}{dx} = 0,$$

the solution of which, satisfying the boundary conditions

$$t_{00} = 1 \text{ at } x = 0, \; t_{00} = 0 \text{ at } x \to \infty,$$

has the form

$$t_{00} = 1 - \frac{3}{\Gamma(1/3)} \int_0^x e^{-\xi^3} d\xi, \tag{14}$$

where $\Gamma(\alpha)$ is a gamma function.

Using Eq. (14), we find that the local thermal flux from the cylinder is equal to

$$q = -\lambda \left(\frac{\partial t_{00}}{\partial \tilde{r}} \right)_{\tilde{r}=a} (\tilde{T}_w - \tilde{T}_\infty) = 0{,}85 \frac{\lambda}{d} (\mathrm{Pr} \mathrm{Re}_p^2)^{1/3} \frac{\sin^{1/2} \varphi}{\left[\int_0^{\varphi} \sin^{1/2} \chi d\chi \right]^{1/3}}.$$

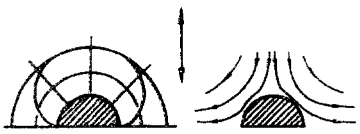

Fig. 1

Then the expression for the local dimensionless heat liberation coefficient, calculated over cylinder diameter, takes on the form

$$Nu = 0.85\,Pr^{1/3}Re_p^{2/3}\frac{\sin^{1/2}\varphi}{\left[\int\limits_0^{\varphi}\sin^{1/2}\chi d\chi\right]^{1/3}}, \tag{15}$$

where $Re_p = Ud/\nu$ is the Reynolds number calculated from the mean square pulsation velocity ($U = B/\sqrt{2}$) and cylinder diameter.

The distribution of the local heat exchange coefficient along the cylinder surface as calculated by Eq. (15) is shown in Fig. 1. Also shown is the structure of secondary flows, formed near the cylinder with low-frequency oscillations, taken from [4]. As follows from Fig. 1, the heat exchange coefficient distribution over the cylinder surface is not uniform. Thus, at the point where the secondary flows are incident on the cylinder surface, which point lies on a line coinciding with the oscillation direction, the local heat exchange coefficient reaches its maximum value, decreasing with movement toward the point of departure of the secondary flows from the surface, at which point Nu = 0. A similar distribution for high-frequency cylinder oscillations was first obtained in [1].

Using Eq. (15), we will calculate an average over the surface for the dimensionless heat exchange coefficient $\overline{Nu} = 0.73Pr^{1/3}Re_p^{2/3}$, or

$$\overline{Nu} = 0.73\,(Ud/\sqrt{D\nu})^{2/3}. \tag{16}$$

Thus, with low-frequency cylinder oscillations the dimensionless heat exchange coefficient is proportional to the pulsation velocity as $U^{2/3}$, to the cylinder size as $d^{2/3}$, inversely proportional to the thermal diffusivity as $D^{-1/3}$, and to the kinematic viscosity as $\nu^{-1/3}$, and in contrast to the case of high-frequency oscillation, does not depend on frequency.

In [5] results were presented from an experimental investigation of heat exchange with a cylinder $1.98 \cdot 10^{-2}$ mm in diameter, oscillating in a highly viscous liquid (fuel oil, auto oil, spindle oil). The oscillation frequency was varied over the range 1.7-27.0 Hz. Kinematic viscosity was $\nu = (66.2\text{-}1.28)\cdot 10^{-4}$ m²/sec, Prandtl number $Pr = (150\text{-}1.4)\cdot 10^2$, and amplitude of cylinder displacement $S = (0.25\text{-}2.0)\cdot 10^{-2}$ m, i.e., the majority of the assumptions made in the theoretical solution of the problem were fulfilled, with the exception of small amplitude of the medium displacement.

The experimental formula of these authors had the form

$$\overline{Nu} = 0.146Re_p^{0.67}Pr^{0.51},$$

i.e., the exponent of the Reynolds number practically coincides with the theoretical value, although the dependence on Prandtl number is stronger than theory predicts.

In [6], on the basis of the results of [5] and their own experimental results, the authors recommended the following empirical expression for calculating heat exchange of a cylinder with low-frequency oscillations:

$$\overline{Nu} = 0.182\,Re_p^{0.64}Pr^{0.32}, \tag{17}$$

Here the functional connection between the dimensionless heat exchange coefficient and the remaining parameters of the process practically coincides with theory.

Equation (17) was also obtained in a recent experimental study [7].

We must note a certain elevation in the values of the heat exchange coefficient calculated with the analytical Eq. (16). This disagreement is in our opinion primarily due to disruption of the condition of low oscillation amplitude, and also to the fact that in solution of the thermal part of the problem a velocity profile obtained by the Ozeen method was used, this method giving elevated velocity values near the cylinder surface due to an increased contribution of convective terms to the motion equation, and thus elevating the values of the heat exchange coefficient.

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EFFECT OF THERMAL DIFFUSION ON FREE CONVECTION OF A BINARY MIXTURE IN A CAVITY WITH A SQUARE CROSS-SECTION

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It is well known that the phenomenon of thermal diffusion can greatly affect the convective stability of a binary mixture consisting of nonreacting components [1]. Convective stability of equilibrium in a liquid binary mixture in a planar horizontal layer was studied in [2-9]. In [3-7], a hysteresis loop was obtained in Benard's problem for a two-component fluid and in [3-5] this problem was also studied experimentally. The effect of thermal diffusion on the convective stability of equilibrium and convective heat and mass transfer in a vertical gap was studied in [1, 10, 11]. In [12], the effect of thermal diffusion on heat transfer through a boundary layer was studied theoretically and experimentally. Free convection of a binary fluid mixture in an inclined rectangular cavity was investigated in [13].

In this paper, we study numerically free convection of a binary mixture in a square horizontal cylinder taking into account thermal diffusion. We examine lateral heating. It is assumed that thermal diffusion is the only reason for the appearance of a concentration inhomogeneity. The investigation is carried out for gas mixtures and aqueous solutions of salts. It is shown that in the presence of weak convection the normal thermal diffusion can double the convective velocity, while anamolous thermal diffusion can decrease it. For Rayleigh numbers of the order of 10^3, a vertical component appears in the concentration gradient at the center of the cavity. For anamalous thermal diffusion, it turns out that the maximum value of the stream function is not a unique function of the Rayleigh number (hysteresis is observed). For Rayleigh numbers exceeding 10^4, the effect of thermal diffusion on convective motion can be neglected.

We will examine an infinite square horizontal cylinder with height a, filled with a binary fluid mixture. The lateral boundaries are impenetrable and have different temperatures T_1 and T_2. The upper and lower boundaries are also impenetrable to matter and have a linear temperature distribution. If there is no convection in the cavity, then the concentration field arising as a result of the Soret effect is nearly linear [14, 15]. The maximum concentration differentials are very small [11], so that we will neglect energy flow caused by the inhomogeneity of the mixture. The Soret coefficient is assumed to be constant. The system of dimensionless equations describing two dimensional motion has the form [1, 16]

$$\frac{\partial\varphi}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial\varphi}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial\varphi}{\partial x}=\Delta\varphi+\mathrm{Gr}\left(\frac{\partial T}{\partial x}+\varepsilon\frac{\partial C}{\partial x}\right),$$

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